1060 – Day 24
Fall 2013

5.3 – Logarithmic Functions

Our goal today is to define & discuss

The inverse of $f(x) = a^x$

Recall: for any positive # a,

$f(x) = a^x$ is called

the exponential function

with base a

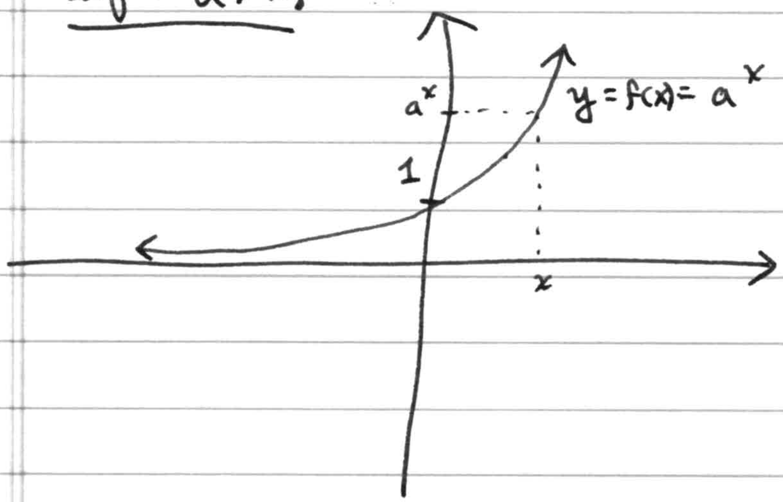
5-10

From Arithmetic, we know a few things

	Eg:
$a^{r+s} = a^r \cdot a^s$	$2^{3+5} = 2^3 2^5$
$a^{r-s} = \dfrac{a^r}{a^s}$	$2^{3-5} = \dfrac{2^3}{2^5}$
$(a^r)^s = a^{r\cdot s}$	$(2^3)^5 = 2^{3\cdot 5}$
$(a\cdot b)^r = a^r \cdot b^r$	$(2\cdot 3)^5 = 2^5 \cdot 3^5$

Also Remember: The graph of $f(x) = a^x$ depends on the # a.

If $a > 1$:



Bigger base a
$\Rightarrow$ a^x **grows faster.**

Exponentials & Logs are important

and formulas are **prettiest**
if the base a is a certain #

$e \approx 2.718...$

Because $2 < e < 3$

e^x grows **faster** than 2^x
and **slower** than 3^x

The Logarithm – our most important inverse

Define: $\log_a(x)$ is the inverse of a^x

That is: $\log_a(x) = \begin{bmatrix} \text{the \# } r \\ \text{s.t. } a^r = x \end{bmatrix}$

so $\log_2(2^b) = b$

and $2^{\log_2(b)} = b$

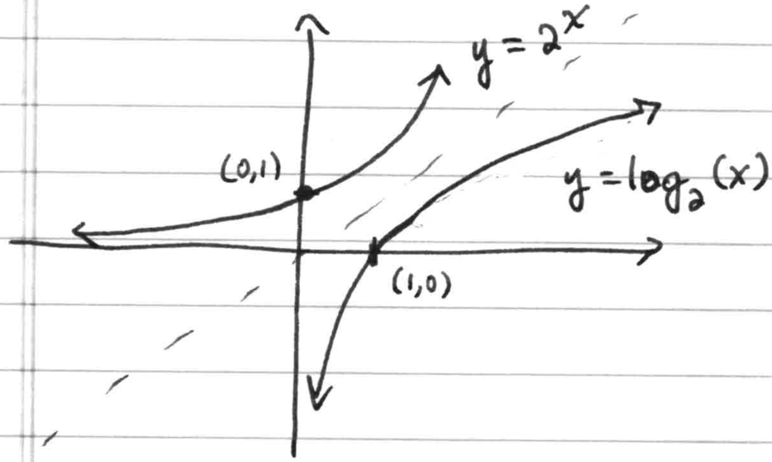
Eg: $\log_{10}(100) = \log_{10}(10^2) = \begin{bmatrix} \text{the \# } r \\ \text{s.t. } 10^r = 10^2 \end{bmatrix}$

$\Rightarrow \log_{10}(100) = 2$

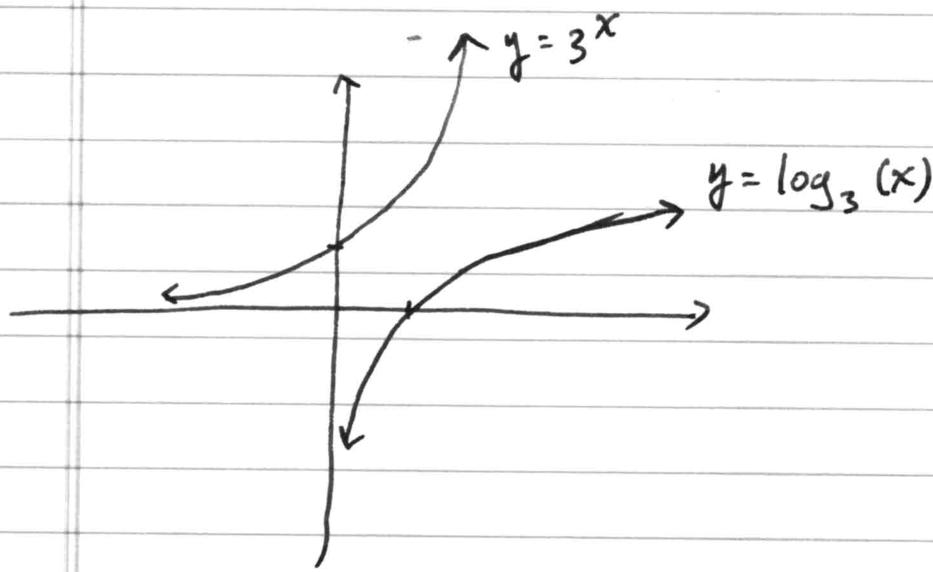
Eg: $\log_2(8) = \log_2(2^3) = \begin{bmatrix} \text{the \# } r \\ \text{s.t. } 2^r = 2^3 \end{bmatrix}$

$\Rightarrow \log_2(8) = 3$

we'll look at a couple of tricks for computing logs,
but first lets look at the graph of $\log_a(x)$



$y = 2^x$ grows <u>slower</u>
$\Rightarrow$
$y = \log_2(x)$
grows <u>faster</u>



$\log_3(x)$ grows slower

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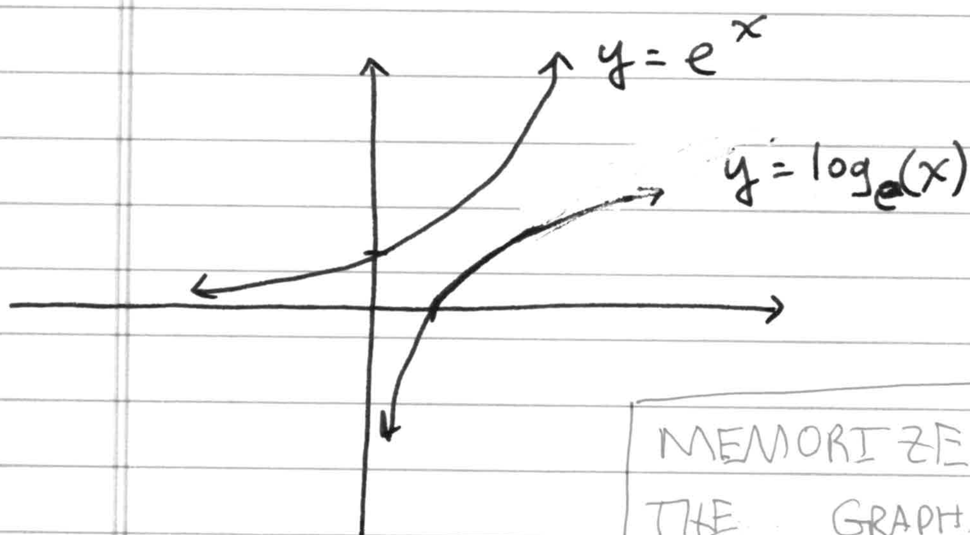
$\log_e(x)$
grows in the middle



MEMORIZE
THE GRAPH OF $\log_e(x)$

e^x is the <u>natural exponential</u> to use in calculus.

$$\boxed{\begin{array}{l}\text{we DEFINE / NAME} \\ \ln(x) = \log_e(x)\end{array}}$$

& call this the <u>natural logarithm</u>.

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Eg: $\ln(e^5) = \log_e(e^5) = 5$

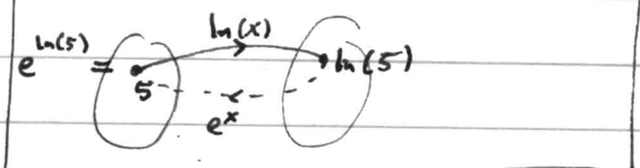
Eg: $\ln\left(\frac{1}{e}\right) = \ln(e^{-1})$

$= \log_e(e^{-1})$

$\ln\left(\frac{1}{e}\right) = -1$

Eg: $e^{\ln(5)} = 5$

Because e^x & $\ln(x)$ are inverses



Remember: $\log_b(b^r) = r$

for ALL #'s r

So $\log_2(2^{x+9}) = x+9$

$\log_3(3^{2x-1}) = 2x-1$

NOTICE $\log_2(x)$ undoes 2^x

But does NOT undo 3^x

we need another property to simplify $\ln(3^{x+2}) = \log_e(3^{x+2})$

$$\boxed{\begin{array}{l}\text{PROPERTY} \\ \quad \text{for all } a, b, \& \ r \\ \qquad \log_b(a^r) = r \cdot \log_b(a)\end{array}}$$

So $\ln(3^{x+2}) = (x+2) \cdot \ln(3)$

[redacted]

why is this true?

want: $\ln(2^3) = 3 \cdot \ln(2)$

Know: $2 = e^{\ln(2)}$

$$2^3 = \left(e^{\ln(2)}\right)^3$$

$$2^3 = e^{3 \cdot \ln(2)}$$

$$\ln(2^3) = \ln\left(e^{3 \cdot \ln(2)}\right)$$

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$$\ln(2^3) = 3 \cdot \ln(2)$$

In general, same trick shows

$$\log_b(a^r) = r \cdot \log_b(a)$$

$\boxed{\text{Why is } \quad 2 = e^{\ln(2)} \quad ?}$

Remember

$$f^{-1}(x) = \left[\begin{array}{l}\text{the } \# r \\ \text{s.t. } \quad x = f(r)\end{array}\right]$$

$\boxed{\text{So}}$ $\quad f\left(f^{-1}(x)\right) = x$

$\boxed{\text{and}}$ $\quad f^{-1}\left(f(x)\right) = x$

when $f(x) = e^{x}$

$f^{-1}(x) = \ln(x) = \log_e(x)$

This means

$$f\left(f^{-1}(2)\right) = e^{\ln(2)}$$

But also

$$f\left(f^{-1}(2)\right) = 2$$

So $\quad 2 = e^{\ln(2)}$

<u>Simplify</u> $e^{2\cdot\ln(3)}$.

$$\boxed{e^{2\cdot\ln(3)} = \left(e^{\ln(3)}\right)^{2} = 3^{2}}$$

<u>Simplify</u> $e^{\frac{\ln(3)}{2}}$.

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$$e^{\frac{\ln(3)}{2}} = e^{\frac{1}{2}\cdot\ln(3)} = \left(e^{\ln(3)}\right)^{\frac{1}{2}}$$

$$\boxed{e^{\frac{\ln(3)}{2}} = 3^{\frac{1}{2}} = \sqrt{3}}$$

<u>Simplify</u> $e^{-3\ln(2)}$.

$$e^{-3\cdot\ln(2)} = \left(e^{\ln(2)}\right)^{-3} = (2)^{-3}$$

$$\boxed{e^{-3\cdot\ln(2)} = \frac{1}{2^3} = \frac{1}{8}}$$

<u>In general:</u> $\boxed{a^x = e^{x\cdot\ln(a)}}$